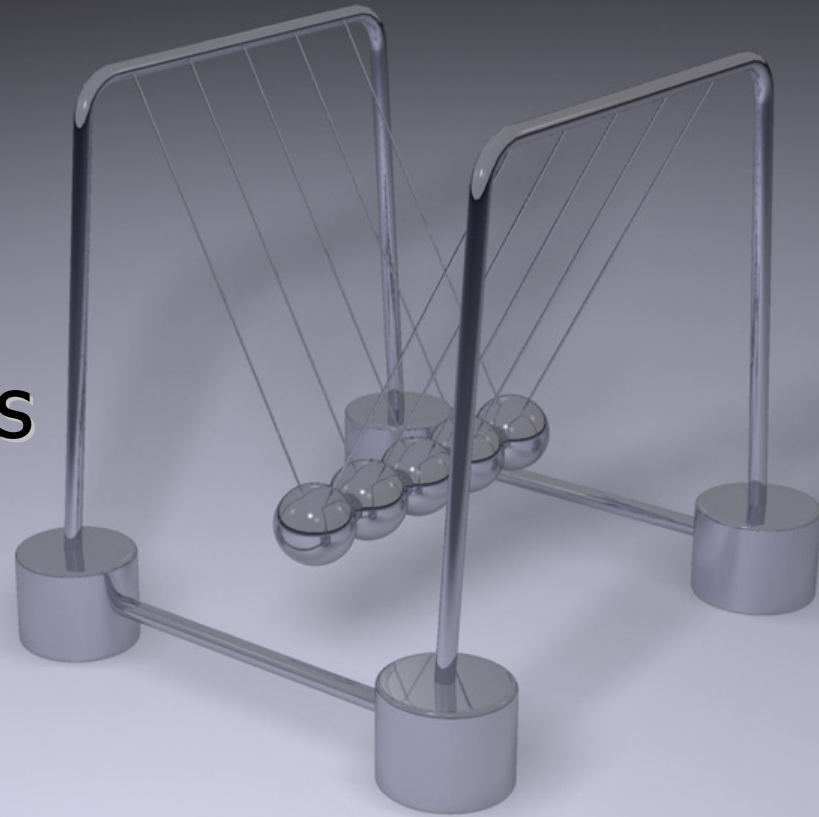


DM2212

Programming Physics



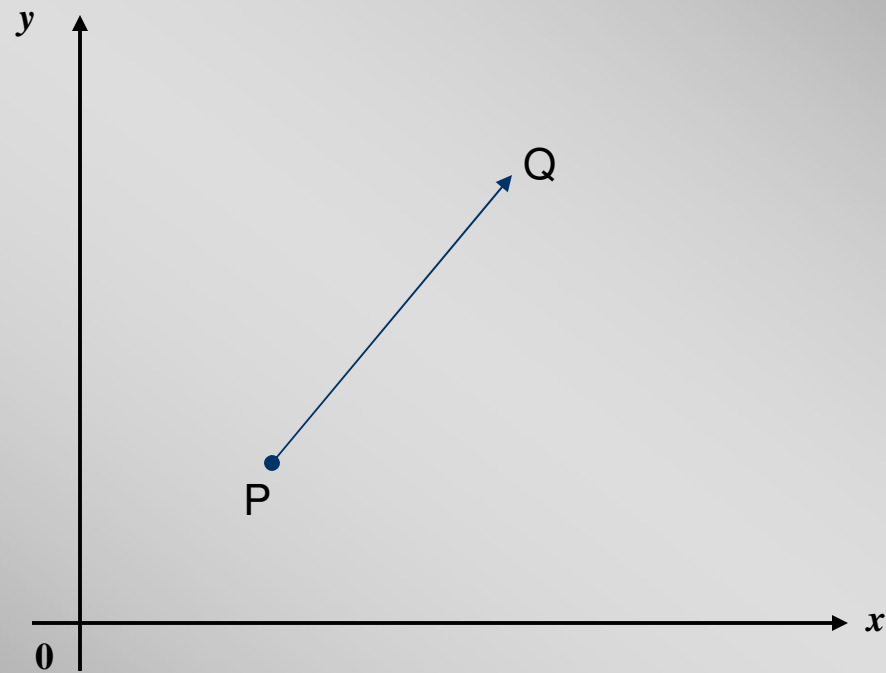
FALLING APPLE^S
MINDPHASE

ML: Dioselin Gonzalez
2007 S1

Vector theory review

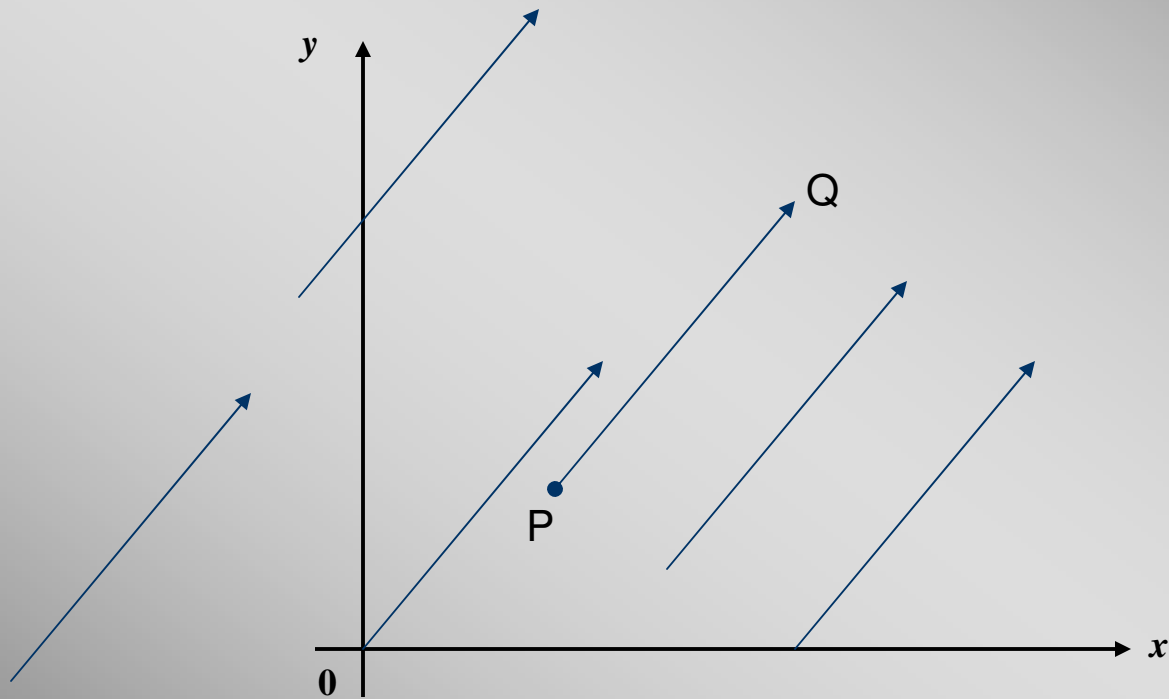


Directed line segment



$$\vec{PQ} \neq \vec{QP}$$

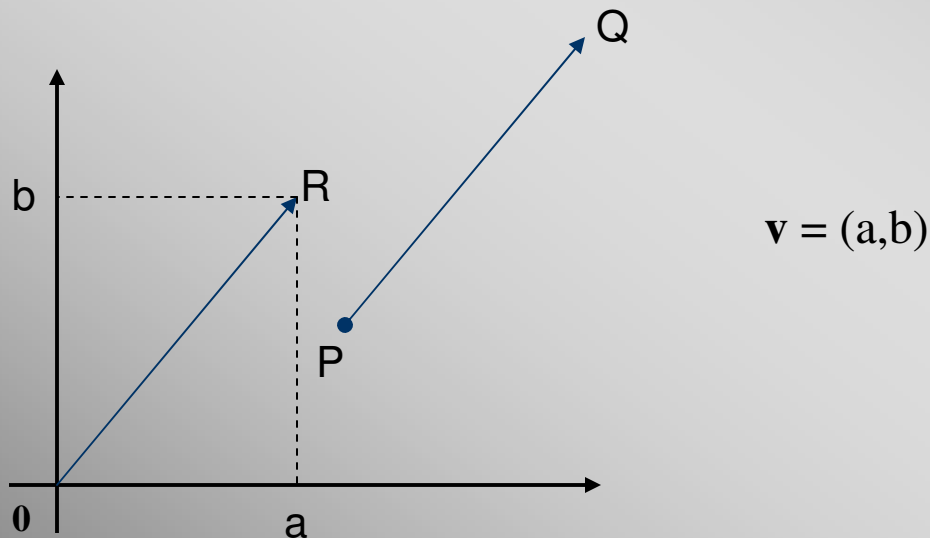
Equivalent directed line segments



Same magnitude and direction

Vector

- Geometric definition:
 - The set of all directed line segments that are equivalent to a given directed segment. Any segment from that set is known as *representative* of the vector.



Vector

- Algebraic definition:
 - A vector \mathbf{v} in \mathbb{R}^3 is an ordered triad of real numbers (x,y,z)
 - x , y and z are known as the *components* of vector \mathbf{v}
 - The *zero vector* is $(0,0,0)$

Both definitions describe the exact same objects!

Vector's magnitude

- Magnitude of $\mathbf{v} = (x, y, z)$

$$|\mathbf{v}| = \sqrt{x^2 + y^2 + z^2}$$

- Unit vector

$$|\mathbf{v}| = 1$$

What's the distance between $P=(x_1, y_1, z_1)$
and $Q=(x_2, y_2, z_2)$?

Vector operations

Let $\mathbf{u}=(x_1,y_1,z_1)$ and $\mathbf{v}=(x_2,y_2,z_2)$ be two vectors and α a real number (scalar)

$$\mathbf{u}+\mathbf{v} = (x_1+x_2, y_1+y_2, z_1+z_2)$$

$$\alpha \mathbf{u} = (\alpha x_1, \alpha y_1, \alpha z_1)$$

Vector's direction

- The direction of vector \mathbf{v} in \mathbb{R}^3 is defined as the direction of unit vector $\mathbf{u} = \mathbf{v} / |\mathbf{v}|$

$$\cos\alpha = \frac{x}{|\mathbf{v}|}$$

$$\cos\beta = \frac{y}{|\mathbf{v}|}$$

$$\cos\gamma = \frac{z}{|\mathbf{v}|}$$

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = ?$$

Dot product

- Let $\mathbf{u}=(x_1,y_1,z_1)$ and $\mathbf{v}=(x_2,y_2,z_2)$ be two vectors

$$\mathbf{u} \cdot \mathbf{v} = x_1x_2 + y_1y_2 + z_1z_2$$

$$\mathbf{u} \cdot \mathbf{v} = |\mathbf{u}| |\mathbf{v}| \cos\varphi$$

\mathbf{u} and \mathbf{v} are parallel if φ is zero or π , and orthogonal (or perpendicular) if ?

<http://www.ies.co.jp/math/java/trig/graphCosX/graphCosX.html>

Cross product

- Let $\mathbf{u}=(a_1,b_1,c_1)$ and $\mathbf{v}=(a_2,b_2,c_2)$ be two vectors

$$\mathbf{u} \times \mathbf{v} = (b_1c_2 - c_1b_2, c_1a_2 - a_1c_2, a_1b_2 - b_1a_2)$$

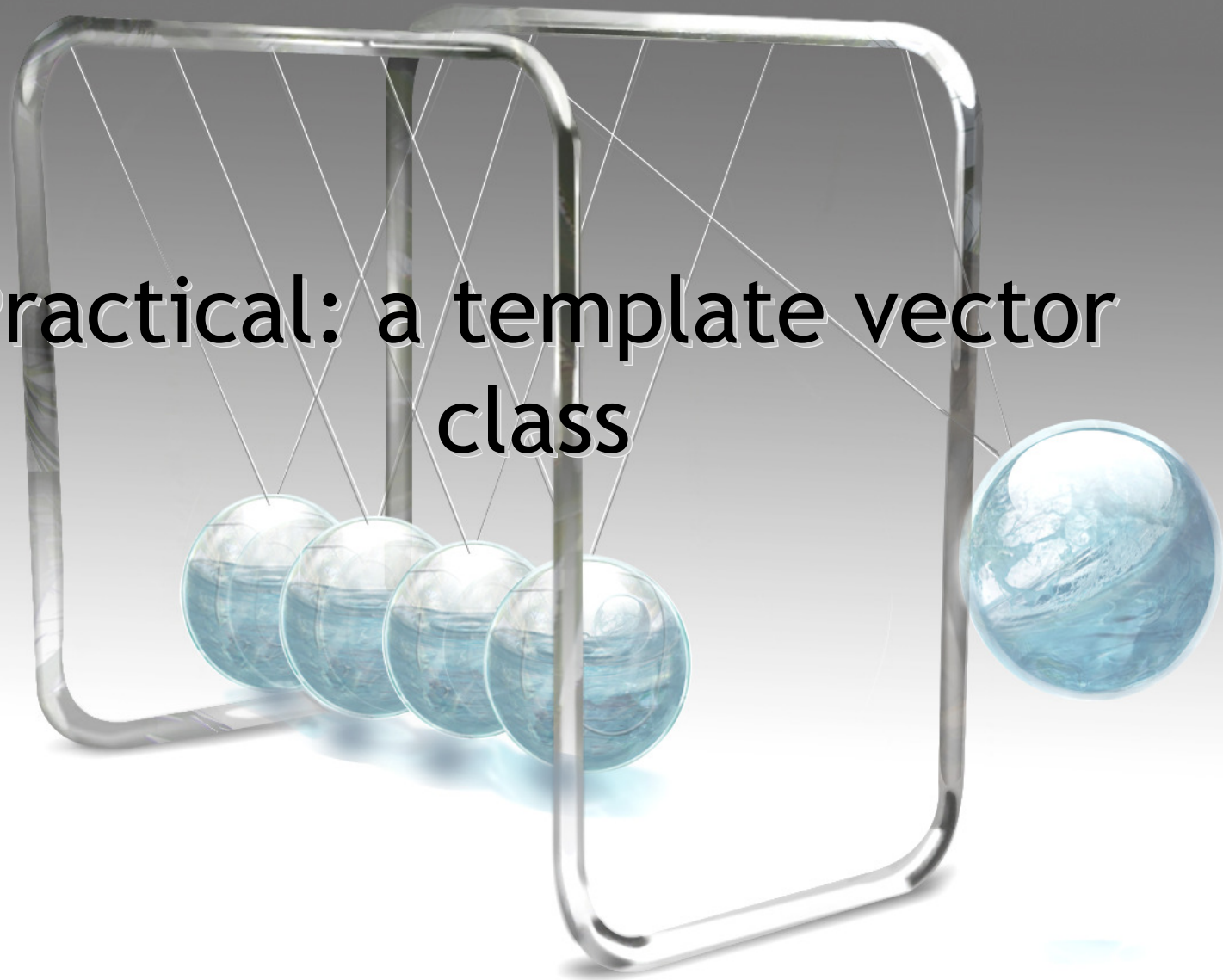
Easier way: determinant notation

$$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = \mathbf{i} \begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix} - \mathbf{j} \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix} + \mathbf{k} \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$$

Cross product theorems

1. $\mathbf{u} \times \mathbf{0} = \mathbf{0} \times \mathbf{u} = \mathbf{0}$
2. $(\mathbf{u} \times \mathbf{v}) = -(\mathbf{v} \times \mathbf{u})$
3. $(\alpha \mathbf{u}) \times \mathbf{v} = \alpha (\mathbf{u} \times \mathbf{v})$
4. $\mathbf{u} \times (\mathbf{v} + \mathbf{w}) = (\mathbf{u} \times \mathbf{v}) + (\mathbf{u} \times \mathbf{w})$
5. $(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w} = \mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})$
6. $\mathbf{u} \cdot (\mathbf{u} \times \mathbf{v}) = \mathbf{v} \cdot (\mathbf{u} \times \mathbf{v}) = \mathbf{0}$
7. If \mathbf{u} and \mathbf{v} are parallel then $\mathbf{u} \times \mathbf{v} = \mathbf{0}$
8. $|\mathbf{u} \times \mathbf{v}| = |\mathbf{u}| |\mathbf{v}| \sin \phi$

Practical: a template vector class



C++ templates

```
template <class identifier> function_declaration;  
template <typename identifier> function_declaration;  
template <class identifier> class_declaration;  
template <typename identifier> class_declaration;
```

```
template <class T>  
T getMax (T a, T b) {  
    return ((a>b) ? a : b);  
}
```

```
void main() {  
    int a,b,c;  
    a = 10; b=15;  
    c = getMax<int>(a,b);  
    //also: c = getMax(a,b);  
}
```

C++ templates

```
template <class T, int N>
class mysequence {
    T memblock [N];
public:
    void setmember (int x, T value);
    T getmember (int x);
};

void main() {
    mysequence<char, 5>;
    mysequence.setmember (1, 'h');
    mysequence.setmember (2, 'i');
    mysequence.setmember (1, 0);

}
```

Today's lab: template vector3 class

- Methods:
 - Copy constructor
 - Magnitude
 - Normalize
 - Direction
- Operators and functions:
 - * and / (multiplication and division by scalar)
 - * (dot product)
 - + , - , =
 - Cross product
 - Angle between two vectors

References

- <http://www.cplusplus.com/doc/tutorial/templates.html>
- <http://www.devarticles.com/c/a/Cplusplus/The-Mighty-Cplusplus-Template/>